

Strange Quark, Tachyonic Gluon Masses and $|V_{us}|$ from Hadronic Tau Decays

Stephan Narison

Laboratoire de Physique Théorique et Astrophysiques
Université de Montpellier II
Place Eugène Bataillon
34095 - Montpellier Cedex 05, France

Abstract

We apply, to the new OPAL data on $V+A$ strange spectral functions, a suitable combination of QCD spectral sum rules directly sensitive to the combination of strange quark (m_s) and tachyonic gluon (λ^2) masses. Using the mean value of λ^2 from a sum rule analysis of different channels, we deduce the invariant strange quark mass $\hat{m}_s = (106^{+33}_{-37})$ MeV to order α_s^3 , which leads to the running mass $\overline{m}_s(2 \text{ GeV}) = (93^{+29}_{-32})$ MeV. We also obtain an interesting though less accurate estimate of the CKM angle: $|V_{us}| = 0.217 \pm 0.019$.

1 Introduction

The determination of the strange quark running mass is of prime importance for low-energy phenomenology, for CP-violation and for SUSY-GUT or some other model-buildings. Since the advent of QCD, where a precise meaning for the definition of the running quark masses within the \overline{MS} -scheme [1] has been provided, a large number of efforts have been devoted to the determinations of the strange quark mass ¹ using QCD spectral sum rules ² à la SVZ [5], in the pseudoscalar [6], the scalar [7], the e^+e^- [8, 9] channels, tau-decay data [10, 11, 12] and lattice simulations [13], while some bounds have been also derived from the positivity of the spectral functions and from the extraction of the quark condensate [14, 3].

In the following, we propose a new method for determining the combination of the strange quark mass and tachyonic gluon mass (λ^2), which will allow us to test the effect of this new λ^2 -term not present in the standard OPE. This λ^2 -term has been introduced in [16] in order to mimic the UV renormalon contribution in the resummation of the PT series, where it is expected to replace the uncalculated infinite number of terms of the PT series. Its value has been estimated from e^+e^- into hadrons data [17, 18] to be: $(\alpha_s/\pi)\lambda^2 \simeq -(0.06 \pm 0.03) \text{ GeV}^2$, while the pseudoscalar channel and a fit of the lattice data in the x -space for the (pseudo)scalar and V+A channels leads to $(\alpha_s/\pi)\lambda^2 \simeq -(0.12 \pm 0.06) \text{ GeV}^2$. In the following, we shall consider the average of the previous estimates:

$$d_2 \equiv (\alpha_s/\pi)\lambda^2 \simeq -(0.07 \pm 0.03) \text{ GeV}^2, \quad (1)$$

which is almost scale independent. One should notice that, the effect of this term which is relatively negligible does not perturb and, in some cases, improves the well-established existing sum rules results [18, 17, 19] and the precise determination of α_s from τ -decays [17, 20] obtained originally in [21]. It also solves [17] the sum rule puzzle scales noticed by [22] in the pseudoscalar pion and gluonia channels, where the scales are much larger than the one of the ρ meson. We shall further study the effect of d_2 in the determination of the strange quark mass from tau decays. In previous analysis of this channel [10, 11, 12], inspired from the first analysis of tau-decay data [21], the λ^2 -effect, which is flavour independent, is absent, to leading order of PT, as the authors work with the difference of the spectral functions in the $\bar{u}d$ and $\bar{u}s$ channels (so-called flavour breaking sum rules). However, the price to pay is the large cancellation between the two different spectral functions, implying a large error bar in the final result. Some other eventual problems appearing in these analysis, are the decrease of the value of the strange quark mass output (absence of stability in the number of moments) and the deterioration of the convergence of the OPE for increasing dimension of the moments, and the non-trivial separation of the spin zero and one parts of the spectral functions in some of the analysis. In this paper, we avoid these problems by working with a given flavoured $\bar{u}s$ spectral function involving the sum of the spin zero and one mesons, but in the same time, our analysis will be affected by λ^2 . This spectral function has been measured by ALEPH, OPAL and CLEO [23], and more recently by OPAL [24].

¹For reviews, see e.g. [2, 3, 4].

²For a review see e.g. [4].

2 The sum rules and the QCD expression

We shall be concerned with the two-point correlator:

$$\Pi_{\mu\nu}^{V+A}(q^2) \equiv i \int d^4x e^{iqx} \langle 0 | \mathcal{T} J_\mu(x) J_\nu^\dagger(0) | 0 \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_{V+A}^{(1+0)}(q^2) + q^2 g_{\mu\nu} \Pi_{V+A}^{(0)}(q^2) \quad (2)$$

built from the charged local weak current:

$$J_\mu = \bar{u} \gamma_\mu (1 - \gamma_5) s. \quad (3)$$

The upper indices 0, 1 refer to the corresponding spin of hadrons entering into the spectral function. Following SVZ [5], the correlator can be approximated by:

$$\Pi_{V+A}(Q^2) \simeq \sum_{d \geq 0} \frac{\mathcal{O}_{2d}}{(Q^2)^d} \quad (4)$$

where $\mathcal{O}_{2d} \equiv C_{2d} \langle \mathcal{O}_{2d} \rangle$ is the short hand-notation of the QCD non-perturbative condensates $\langle \mathcal{O}_{2d} \rangle$ of dimension $D \equiv 2d$ and its associated perturbative Wilson coefficient C_{2d} ; $q^2 \equiv -(Q^2 > 0)$ is the momentum transfer. The spectral function $(v + a)$:

$$\frac{1}{\pi} \text{Im} \Pi_{V+A} \equiv \frac{1}{2\pi^2} (v + a). \quad (5)$$

has been measured using τ -decay data, via:

$$\begin{aligned} v_1/a_1 &= \frac{M_\tau^2}{6|V_{us}|^2 S_{ew}} \left(1 - \frac{t}{M_\tau^2}\right)^{-2} \left(1 + \frac{2t}{M_\tau^2}\right)^{-1} \frac{B(\tau \rightarrow (V/A)^{(S=-1, J=1)} + \nu_\tau)}{B(\tau \rightarrow e^- \bar{\nu}_e + \nu_\tau)} \frac{1}{N_{V/A}} \frac{dN_{V/A}}{dt} \\ v_0/a_0 &= \frac{M_\tau^2}{6|V_{us}|^2 S_{ew}} \left(1 - \frac{t}{M_\tau^2}\right)^{-2} \frac{B(\tau \rightarrow (V/A)^{(S=-1, J=0)} + \nu_\tau)}{B(\tau \rightarrow e^- \bar{\nu}_e + \nu_\tau)} \frac{1}{N_{V/A}} \frac{dN_{V/A}}{dt}, \end{aligned} \quad (6)$$

where OPAL has used $|V_{us}| = 0.2196 \pm 0.0023$ [25], $M_\tau = 1776.9_{-0.27}^{+0.31}$ MeV [26] and $S_{ew} = 1.0194 \pm 0.0040$ [27].

For a pedagogical purpose, we write the QCD expression of the $V+A$ correlator to leading order in α_s and m_s (see e.g. [21, 4]), and including the leading new λ^2 -tachyonic gluon term [17]:

$$\Pi_{V+A}^{(0+1)}(Q^2) = \frac{1}{2\pi^2} \left\{ -\ln \frac{Q^2}{\nu^2} - \frac{\mathcal{O}_2^0}{Q^2} + \frac{\mathcal{O}_4^0}{Q^4} + \frac{\mathcal{O}_6^0}{Q^6} \right\}, \quad (7)$$

with obvious notations, where:

$$\begin{aligned} \mathcal{O}_2^0 &= 3m_s^2 + \frac{\alpha_s}{\pi} \lambda^2 \\ \mathcal{O}_4^0 &= 4\pi^2 (m_s \langle \bar{s}s \rangle + m_u \langle \bar{u}u \rangle) + \frac{\pi}{3} \langle \alpha_s (G_{\mu\nu}^a)^2 \rangle \\ \mathcal{O}_6^0 &= \frac{256\pi^3}{81} \alpha_s \langle \bar{u}u \rangle^2 \end{aligned} \quad (8)$$

Its inverse Laplace transform sum rule (LSR) reads [5, 28, 29, 4]:

$$\mathcal{L}_0 \equiv \int_0^{t_c} dt e^{-t\tau} \frac{1}{\pi} \text{Im} \Pi_{V+A}^{(0+1)} = \frac{\tau^{-1}}{2\pi^2} \left\{ 1 - e^{-t_c\tau} - \mathcal{O}_2^0 \tau + \mathcal{O}_4^0 \tau^2 + \frac{1}{2} \mathcal{O}_6^0 \tau^3 \right\}, \quad (9)$$

from which, one can derive, to leading order in α_s and m_s , the LSR:

$$\mathcal{L}_1 \equiv -\frac{d}{d\tau}\mathcal{L}_0 \equiv \int_0^{t_c} dt t e^{-t\tau} \frac{1}{\pi} \text{Im}\Pi_{V+A}^{(0+1)} = \frac{\tau^{-2}}{2\pi^2} \left\{ 1 - (1 + t_c\tau)e^{-t_c\tau} - \mathcal{O}_4^0\tau^2 - \mathcal{O}_6^0\tau^3 \right\}, \quad (10)$$

and the leading order FESR [30, 4]:

$$\begin{aligned} \mathcal{M}_0 &\equiv \int_0^{t_c} dt \frac{1}{\pi} \text{Im}\Pi_{V+A}^{(0+1)} = \frac{t_c}{2\pi^2} \left\{ 1 - \frac{\mathcal{O}_2^0}{t_c} \right\}, \\ \mathcal{M}_1 &\equiv \int_0^{t_c} dt t \frac{1}{\pi} \text{Im}\Pi_{V+A}^{(0+1)} = \frac{t_c^2}{4\pi^2} \left\{ 1 - 2\frac{\mathcal{O}_4^0}{t_c^2} \right\}. \end{aligned} \quad (11)$$

From these previous sum rules, one can derive the combination of sum rules ³:

$$\begin{aligned} \mathcal{N}_{10} &\equiv \mathcal{L}_0 - \tau\mathcal{L}_1 \equiv \int_0^{t_c} dt (1 - t\tau) e^{-t\tau} \frac{1}{\pi} \text{Im}\Pi_{V+A}^{(0+1)} \\ \mathcal{S}_{10} &\equiv \mathcal{M}_0 - \frac{2}{t_c}\mathcal{M}_1 \equiv \int_0^{t_c} dt \left(1 - 2\frac{t}{t_c} \right) \frac{1}{\pi} \text{Im}\Pi_{V+A}^{(0+1)}, \end{aligned} \quad (12)$$

which are sensitive, to leading order, to m_s^2 and λ^2 . Unlike the individual sum rules, these combinations of sum rules are less sensitive to the high-energy tail of the spectral functions (effect of the t_c threshold). Then, one may expect that they are more accurate than the former. However, this accuracy may not be comparable with the one of the τ -decay like-sum rules where threshold effect suppresses completely the effects near the real axis.

We shall also work with the ratio of moments:

$$\mathcal{R}_{10} \equiv 2\frac{\mathcal{M}_1}{\mathcal{M}_0}, \quad (13)$$

which will also be useful for testing the duality between the LHS (experiment) and RHS (QCD theory).

3 QCD corrections and RGI parameters

In order to account for the radiative corrections, one introduces the expressions of the running coupling and masses.

- To three-loop accuracy, the running coupling can be parametrized as [21, 4]:

$$\begin{aligned} a_s(\nu) &= a_s^{(0)} \left\{ 1 - a_s^{(0)} \frac{\beta_2}{\beta_1} \log \log \frac{\nu^2}{\Lambda^2} \right. \\ &\quad \left. + \left(a_s^{(0)} \right)^2 \left[\frac{\beta_2^2}{\beta_1^2} \log^2 \log \frac{\nu^2}{\Lambda^2} - \frac{\beta_2^2}{\beta_1^2} \log \log \frac{\nu^2}{\Lambda^2} - \frac{\beta_2^2}{\beta_1^2} + \frac{\beta_3}{\beta_1} \right] + \mathcal{O}(a_s^3) \right\}, \end{aligned} \quad (14)$$

with:

$$a_s^{(0)} \equiv \frac{1}{-\beta_1 \log(\nu/\Lambda)} \quad (15)$$

³A sum rule similar to \mathcal{N}_{10} has been used for the first time in the pseudoscalar channel for testing the size of the $SU(3)$ breakings in the kaon PCAC relation [31].

and β_i are the $\mathcal{O}(a_s^i)$ coefficients of the β -function in the \overline{MS} -scheme, which read for three flavours [4]:

$$\beta_1 = -9/2, \quad \beta_2 = -8, \quad \beta_3 = -20.1198. \quad (16)$$

- The expression of the running quark mass in terms of the invariant mass \hat{m}_i is [1, 4]:

$$\begin{aligned} \overline{m}_i(\nu) = \hat{m}_i(-\beta_1 a_s(\nu))^{-\gamma_1/\beta_1} & \left\{ 1 + \frac{\beta_2}{\beta_1} \left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_2}{\beta_2} \right) a_s(\nu) \right. \\ & \left. + \frac{1}{2} \left[\frac{\beta_2^2}{\beta_1^2} \left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_2}{\beta_2} \right)^2 - \frac{\beta_2^2}{\beta_1^2} \left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_2}{\beta_2} \right) + \frac{\beta_3}{\beta_1} \left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_3}{\beta_3} \right) \right] a_s^2(\nu) + 1.95168 a_s^3(\nu) \right\}, \end{aligned} \quad (17)$$

where γ_i are the $\mathcal{O}(a_s^i)$ coefficients of the quark-mass anomalous dimension, which read for three flavours [4]:

$$\gamma_1 = 2, \quad \gamma_2 = 91/12, \quad \gamma_3 = 24.8404. \quad (18)$$

- To order α_s^3 , the perturbative expression of the correlator reads, in terms of the running coupling evaluated at $Q^2 = \nu^2$ (see e.g. [21, 4]):

$$-Q^2 \frac{d}{dQ^2} \Pi_{V+A}(Q^2) = \frac{1}{2\pi^2} \left\{ 1 + \left(a_s \equiv \frac{\alpha_s(Q^2)}{\pi} \right) + 1.6398 a_s^2 + 6.3711 a_s^3 + \dots \right\}, \quad (19)$$

- The $D = 2$ contribution reads to order α_s^3 , in terms of the running mass and by including the λ^2 term [21, 17, 4, 12]:

$$Q^2 \Pi_{V+A}^{(D=2)}(Q^2) \simeq -\frac{1}{2\pi^2} \left\{ a_s \lambda^2 + 3\overline{m}_s^2 \left(1 + 2.333 a_s + 19.58 a_s^2 + 202.309 a_s^3 + \mathcal{O}(a_s^4) \right) \right\}. \quad (20)$$

The coefficient of the a_s^4 term has been estimated [12] using PT optimisation schemes arguments to be $K \simeq 2276 \pm 200$. In our approach its effect like all unknown higher order terms will be mimicked by the λ^2 -term present in the $D = 4$ contribution given below.

- The $D = 4$ contributions read [21, 4]:

$$\begin{aligned} Q^4 \Pi_{V+A}^{(D=4)}(Q^2) \simeq & 2 \left\{ \frac{1}{12\pi} \left(1 - \frac{11}{18} a_s \right) \langle \alpha_s G^2 \rangle \right. \\ & + (1 - a_s) \langle m_u \bar{u}u + m_s \bar{s}s \rangle + \frac{4}{27} a_s \langle m_s \bar{u}u + m_u \bar{s}s \rangle \\ & + \frac{1}{4\pi^2} \left(-\frac{12}{7} a_s^{-1} + 1 \right) \overline{m}_s^4 - \frac{1}{28\pi^2} \left[1 - \left(\frac{65}{6} - 16\zeta(3) \right) a_s \right] \overline{m}_s^4 \\ & \left. + \frac{1}{4\pi^2} \left(-\frac{25}{3} + 4\zeta(3) \right) m_s^2 a_s \lambda^2 \right\}, \end{aligned} \quad (21)$$

where the last term is due to the λ^2 term [17], and $\zeta(3)=1.202\dots$. We shall use as input $\Lambda_3 = (375 \pm 25)$ MeV for three flavours, the value of $a_s \lambda^2$ in Eq. (1) and [32, 4, 33]:

$$\begin{aligned} (m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle &= -2m_\pi^2 f_\pi^2 \\ (m_s + m_u) \langle \bar{s}s + \bar{u}u \rangle &\simeq -2 \times 0.7 m_K^2 f_K^2 \\ \langle \alpha_s G^2 \rangle &\simeq (0.07 \pm 0.01) \text{ GeV}^4 \\ \rho \alpha_s \langle \bar{u}u \rangle^2 &\simeq (5.8 \pm 0.9) \times 10^{-4} \text{ GeV}^6, \end{aligned} \quad (22)$$

where: $f_\pi = 93.3$ MeV, $f_K = 1.2f_\pi$ and we have taken into account a possible violation of kaon PCAC as suggested by the QSSR analysis [31, 4]; $\rho \simeq 2 - 3$ is the measures of the deviation from the vacuum saturation estimate of the four-quark condensates.

- FESR including radiative corrections can be deduced from the previous expressions of the correlator using either the Laplace or the Gaussian transforms [30, 4]. For the FESR, the most dominant contributions induced by the radiative corrections to the unit operator is the dimension-two terms:

$$\delta\mathcal{O}_2 = - \left(\frac{t_c}{2\pi^2} \right) \left(\frac{\beta_1}{4} \right) a_s^2, \quad (23)$$

which adds to the contribution of the terms in Eq. (20). One can check that this term remains a correction of the λ^2 contribution in this previous equation. Some other corrections induced by the log-dependence of the running terms taken into account in the numerical analysis remains also tiny corrections.

4 Phenomenological analysis

We parametrize the spectral function by using the most recent OPAL data discussed in a previous section for t until M_τ^2 . In so doing, we parametrize the data using standard polynomials fits which delimit the domain spanned by the error bars of the data. At the level of accuracy of about 30%, where m_s will be determined, the inclusion of the correlations of each data points are not necessary. In fact, we have done similar methods in parametrizing the τ and e^+e^- data in the most accurate determination of the $g - 2$ of the muon, where the error in the estimate is at the level of 1% [34]. In this accurate example, our result and the errors agreed quite well with ones where the correlations among different data points have been taken into account [35]. For t above M_τ^2 , we add the QCD step function:

$$\frac{1}{\pi} \text{Im}\Pi_{V+A}(t \geq M_\tau^2) \simeq \theta(t - M_\tau^2) \frac{1}{2\pi^2} \left\{ 1 + a_s(t) + 1.6398a_s^2(t) - 10.284a_s^3(t) \right\}, \quad (24)$$

which is consistent with the data at $t = M_\tau^2$.

Test of duality

In principle, the value of the t_c -cut of the FESR integrals is a free parameter. We fix its optimal value by looking for the region where the phenomenological and QCD sides of the ratio of moments in Eq. (13) are equal. We present this analysis in Fig. 1, by showing the value of t_c predicted by the sum rule versus t_c and by comparing the result with the exact solution $t_c = t_c$ for all values of t_c . From Fig. 1, one can see that the upper values of the data points provide stronger constraints on t_c than their central value. Considering this stronger constraint, we deduce, that QCD duality is best obtained at:

$$t_c \simeq M_\tau^2, \quad (25)$$

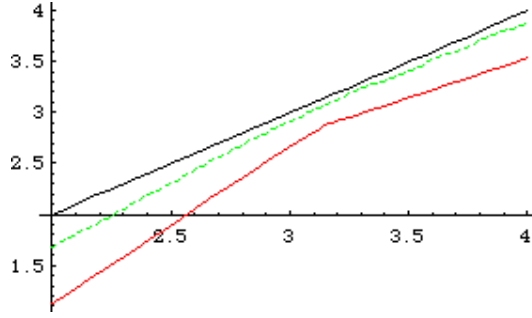


Figure 1: FESR prediction of t_c versus t_c . The green curve corresponds the central value of the data; the red one to the larger values of the data; the black continuous line is the exact solution $t_c = t_c$. The curves correspond to the value $\hat{m}_s = (56 - 145)$ MeV and giving $a_s\lambda^2 = -0.07$ GeV².

where one expects to get the optimal value of m_s from FESR. In order to get this number, we have used the value of the invariant mass $\hat{m}_s = (56 - 145)$ MeV, which is a tiny correction in this duality test analysis. Once we have fixed the value of t_c where the best duality from the two sides of FESR has been obtained, we can now estimate some other observables.

Estimate of \hat{m}_s versus λ^2

We can, in principle, estimate \hat{m}_s using the combinations of LSR or/and FESR in Eq. (12). A sample of analysis is given in Fig. 2 for the LSR and in Fig. 3 from the FESR:

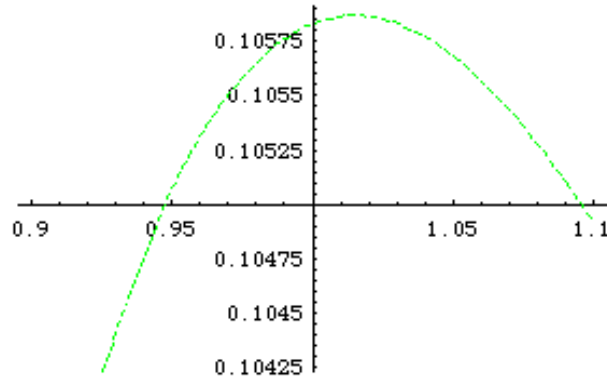


Figure 2: LSR prediction of \hat{m}_s in GeV versus the LSR variable τ in GeV⁻² for $-a_s\lambda^2 = 0.07$ GeV², using the central value of the data and fixing $t_c = 3.15$ GeV².

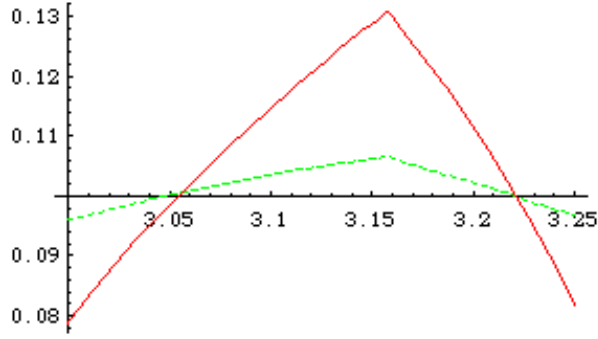


Figure 3: FESR prediction of \hat{m}_s in GeV versus t_c in GeV^2 for $-a_s\lambda^2 = 0.07 \text{ GeV}^2$. The green curve corresponds to the central value of the data; the red one to the larger values of the data.

Table 1: Estimated value of \hat{m}_s versus λ^2

$-a_s\lambda^2$ in GeV^2	\hat{m}_s in MeV
0.02	$53 \pm 38 \pm 8$
0.04	$79 \pm 29 \pm 8$
0.06	$98 \pm 24 \pm 9$
0.07	$106 \pm 23 \pm 9$
0.08	$114 \pm 21 \pm 9$
0.10	$128 \pm 19 \pm 11$
0.12	$140 \pm 18 \pm 11$
0.15	$157 \pm 16 \pm 11$

The first error is due to the data, the second one to Λ . The error due to the choice of t_c around the duality region is quite small of about 3 MeV as can be seen in Fig. 3.

- In Fig. 2, we give the value of \hat{m}_s versus the LSR variable τ at given $t_c = M_\tau^2$. One can notice that, though the “optimal” estimate looks reasonable, it is obtained at the value of τ of about 1 GeV^{-2} , where, at this scale, the PT series of the m_s^2 coefficient behaves badly rendering this result inaccurate. Therefore, we do not consider this result in our final estimate. One should notice that the size of this optimization scale is typical for the LSR, as the exponential amplifies the low energy contribution to the spectral function. This is not the case of the FESR which acts in the opposite region of the spectral function.

- In Fig. 3, we give the value of \hat{m}_s from FESR versus the t_c variable. Optimal estimate is obtained for the t_c -value where the duality between the two sides of FESR is the best. One can notice that unlike the case of LSR, FESR results are obtained at $t_c = 3.15 \text{ GeV}^2$, where the PT series in the m_s^2 make sense.
- Results for different values of λ^2 in the range given in Eq. (1) are shown in Table 1. Considering the mean value of λ^2 from Eq. (1), we deduce from Table 1, the estimate:

$$\hat{m}_s \simeq (106_{-37}^{+33}) \text{ MeV} \quad \text{for} \quad a_s \lambda^2 \simeq -(0.07 \pm 0.03) \text{ GeV}^2 . \quad (26)$$

Using the relation in Eq. (17):

$$\overline{m}_s(2 \text{ GeV}) \simeq 0.876 \hat{m}_s , \quad (27)$$

we translate the result on \hat{m}_s into the value of the running mass at 2 GeV to order α_s^3 :

$$\overline{m}_s(2 \text{ GeV}) \simeq (93_{-32}^{+29}) \text{ MeV} . \quad (28)$$

- This result can be compared with the recent determinations from flavour breaking-sum rule in tau-decays [11, 12], which is not affected to leading order by λ^2 :

$$\overline{m}_s(2 \text{ GeV}) \simeq (81 \pm 22) \text{ MeV} , \quad (29)$$

and with the one:

$$\overline{m}_s(2 \text{ GeV}) \simeq (105 \pm 26) \text{ MeV} , \quad (30)$$

deduced from the pion sum rule to order α_s^3 , where the λ^2 term has been added [17, 3, 4]:

$$(\overline{m}_u + \overline{m}_d)(2 \text{ GeV}) \simeq (8.6 \pm 2.1) \text{ MeV} , \quad (31)$$

plus the ChPT ratio $(m_u + m_d)/2m_s = 24.4 \pm 1.5$ [2]. From the previous values, we can deduce the average⁴:

$$\langle \overline{m}_s(2 \text{ GeV}) \rangle \simeq (92 \pm 15) \text{ MeV} , \quad (32)$$

which is comparable with lattice determinations.

- Finally, one can compare this result with the lower bounds obtained from the positivity of spectral functions proposed in [14] and updated to order α_s^3 to be $(71.4 \pm 3.7) \text{ MeV}$ in [4] (the inclusion of λ^2 decreases this value by 5%), and from an independent lower bound of about 80-90 MeV derived in [15] from direct extractions of the quark condensate. Our result in Eq. 28 is compatible with these bounds. However, values of m_s corresponding to $-\lambda^2 \leq 0.04 \text{ GeV}^2$ are less favoured by these bounds and by the previous estimates from other channels.

⁴We have not included in the average the value of m_s from the vector current in [36], which is now under reconsideration.

Estimate of $|V_{us}|$

We can estimate the CKM angle by working with the FESR \mathcal{M}_0 or/and the LSR \mathcal{L}_0 . FESR gives the result:

$$|V_{us}| \simeq 0.215 \pm 0.017 , \quad (33)$$

while the LSR gives:

$$|V_{us}| \simeq 0.219 \pm 0.021 . \quad (34)$$

One can notice that the result is not sensitive to the change of m_s in the range 56 to 145 MeV, which, a posteriori justifies the use of the exponential Laplace sum rule (LSR), though, in the optimal region, the PT series of the m_s^2 contribution behaves badly. We take, as a final result, the arithmetic average of the FESR and LSR results:

$$|V_{us}| \simeq 0.217 \pm 0.019 , \quad (35)$$

which, despite the large error, is an interesting output per se as it shows the consistency of the approach used to get m_s .

5 Conclusions

Our result for the strange quark mass, in Eq. (28), from the strange spectral function of hadronic tau decays shows that the one for non-zero value of the tachyonic gluon mass is in better agreement with the existing determinations of this quantity from τ -decays and pseudoscalar channels and with the lower bounds derived from different sum rules [14, 15, 3, 4] than the one in the theory without a tachyonic gluon.

Our result in Eq. (35) for V_{us} is less accurate than existing determinations but interesting per se for testing the consistency of the whole approach.

Improvements of the results obtained in this paper require more accurate data, which will also help for a much better determination of m_s or/and of the tachyonic gluon mass λ^2 .

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